Constrained Common Invariant Subspace to Descriptor Multiaffine Representation of Rational Parameter Systems

Dongdong Zhao, *Member, IEEE*, Lei Yan, Yu Kang, *Senior Member, IEEE*, Yun-Bo Zhao, *Senior Member, IEEE*, and Shi Yan, *Senior Member, IEEE*

***Abstract*—This article aims to study the geometric control method of the constrained common invariant subspace (CCIS) within the descriptor multiaffine representation (DMAR) of rational parameter systems, which is particularly effective for deriving S-variable linear matrix inequality results. Specifically, novel necessary and sufficient conditions for the existence of CCIS are established, which significantly reduce the required memory storage and address major computational challenges, such that the practical application of CCIS within DMAR of rational parameter systems becomes possible. The efficient CCIS calculation methodology is then applied to DMARs of rational parameter systems, providing sufficient conditions for their reducibility. Furthermore, an H_2 feedback controller synthesis is proposed based on the DMAR model, demonstrating less conservation compared with existing methods. Moreover, the advantages of employing the efficient CCIS method in the design of H_2 feedback controllers are highlighted, showcasing its superiority in enhancing controller performance.**

***Index Terms*— H_2 feedback controller, constrained common invariant subspace (CCIS), descriptor multiaffine representation (DMAR), rational parameter system.**

I. Introduction

The invariant subspace for a single square matrix plays a pivotal role in geometric control theory [1], especially for linear time-invariant systems, facilitating advancements in system analysis and design, including system identification [2], reducibility (or minimality) [3], controllability analysis [4], observability analysis [5], state feedback, and observer design [6]. The generalized concept of common invariant subspace (CIS) has garnered interest due to the fact that complex systems are often characterized by multiple state matrices [7]. However, identifying CIS existence conditions and formulating computational methodologies remain challenging [8].

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Dongdong Zhao, Lei Yan, and Shi Yan are with the School of Information Science and Engineering, Lanzhou University, Lanzhou 73000, China (e-mail: ddzhao@lzu.edu.cn; yanl2021@lzu.edu.cn; yanshi@lzu.edu.cn).

Yu Kang and Yun-Bo Zhao are with the Department of Automation, University of Science and Technology of China, Hefei 230027, China (e-mail: kangduyu@ustc.edu.cn; ybzhao@ustc.edu.cn).

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A practical approach to these challenges involves the study of CIS under specific constraints. To this end, significant strides have been made by introducing the concept of a constrained common eigenvector in our previous work [9]. This concept represents a specialized form of CIS, defined by a 1-D basis vector $\boldsymbol{w}$ in the kernel space of a given matrix C, i.e., $C\boldsymbol{w} = \mathbf{0}$. To advance the understanding of CISs with constraints and higher dimensionality, we have developed the concept of a constrained common invariant subspace (CCIS) [7]. This framework establishes the necessary and sufficient conditions for the existence of CCIS, significantly simplifying the analysis of the system and allowing the design of less conservative H_∞ state-feedback controllers [7].

On the other hand, the descriptor multiaffine representation (DMAR) offers an efficient alternative for modeling parametric uncertainties with lower dimensions than those typically associated with linear fractional representation (LFR) [10]. Moreover, as highlighted by the authors in [10], [11], and [12], DMAR is particularly well suitable for deriving S-variable linear matrix inequality (LMI) results. Despite advancements in DMAR modeling, the existing literature predominantly employs LFR techniques, which involve formulating an LFR for the system before converting it to a DMAR. This approach, as indicated in [10], often results in DMARs with unnecessarily high dimensions due to the specific structure of LFR-based methods. However, the challenge in computing CCIS for n matrices in r dimensions stems from the dramatic increase in memory storage requirements as both n and r grow, which presents significant obstacles for the practical application of CCIS methods to DMAR.

In light of these considerations, this article aims to propose an effective CCIS calculation method and apply it to DMAR of rational parameter systems. Specifically, a new necessary and sufficient condition is established for the existence of a CCIS with significantly less memory storage. This methodology is then applied to DMARs for the first time, where sufficient conditions are developed for their reducibility. Furthermore, an H_2 control methodology is proposed for discrete-time linear parameter-varying (LPV) systems, while the numerical complexity is significantly reduced by using the efficient CCIS computation strategy.

Notation: Let $\mathbb{R}^n$ denote the n-dimensional real column vector space, and $\mathbb{R}^{1\times n}$ represent the corresponding row vector space. The dimensionality of a space $\mathcal{W}$ is denoted by $\dim(\mathcal{W})$. Zero matrices of appropriate size and specified size $m \times n$ are denoted by $\mathbf{0}$ and $\mathbf{0}_{m\times n}$, respectively. Let the time-varying parameters $\theta_1, \ldots, \theta_N$ be collected in $\theta := (\theta_1, \ldots, \theta_N)$. The field of rational functions in variables θ over $\mathbb{R}$ is represented as $\mathbb{R}(\theta)$. The class of $m \times n$ matrices with entries in $\mathbb{R}$ and $\mathbb{R}(\theta)$ are denoted by $\mathbb{R}^{m\times l}$ and $\mathbb{R}^{m\times l}(\theta)$, respectively. The transpose of a matrix A is denoted by $A^\top$, and $\{A\}^s$ stands for the symmetric matrix $\{A\}^s := A + A^\top$. The kernel of a matrix A is defined as $\ker(A) = \boldsymbol{\eta} \in \mathbb{R}^q : A\boldsymbol{\eta} = \mathbf{0}$. For two matrices A and B, the row space of A is denoted by $\mathrm{spanrow}(A)$, which represents the subspace spanned by the row vectors of A. The sum of the row

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spaces of A and B is given by $\text{span}_{\text{row}}(A) + \text{span}_{\text{row}}(B) = \{\boldsymbol{a} + \boldsymbol{b} : \boldsymbol{a} \in \text{span}_{\text{row}}(A), \boldsymbol{b} \in \text{span}_{\text{row}}(B)\}$ [13].

II. Motivation

This section reviews DMAR and CCIS, revealing the essentially necessity to develop an efficient computational method when applying CCIS to DMAR for rational parameter systems.

A. Descriptor Multiaffine Representation

Consider parameter-dependent systems described by

$$x_{k+1} = A_{xx}(\theta)x_k + B_{xu}(\theta)u_k \tag{1a}$$

$$z_k = C_{zx}(\theta)x_k + D_{zu}(\theta)u_k \tag{1b}$$

where $x_k \in \mathbb{R}^{m_x}$, $u_k \in \mathbb{R}^{m_u}$, and $z_k \in \mathbb{R}^{m_z}$ represent the state vector, control input vector, and controlled output vector, respectively, and the matrices $A_{xx}(\theta) \in \mathbb{R}^{m_x \times m_x}(\theta)$, $B_{xu}(\theta) \in \mathbb{R}^{m_x \times m_u}(\theta)$, $C_{zx}(\theta) \in \mathbb{R}^{m_z \times m_x}(\theta)$, and $D_{zu}(\theta) \in \mathbb{R}^{m_z \times m_u}(\theta)$ are rational and continues in θ.

Rewriting the system (1) in matrix form yields $\begin{bmatrix} x_{k+1} \\ z_k \end{bmatrix} = G(\theta) \begin{bmatrix} x_k \\ u_k \end{bmatrix}$, where $G(\theta)$ is a rational parameter matrix (RPM) defined by

$$G(\theta) := \begin{bmatrix} A_{xx}(\theta) & B_{xu}(\theta) \\ C_{zx}(\theta) & D_{zu}(\theta) \end{bmatrix} \in \mathbb{R}^{(m_x+m_z)\times(m_x+m_u)}(\theta). \tag{2}$$

Definition 1. (Multiaffine matrix): A polynomial matrix $M(\theta)$ is said to be multiaffine if it takes the form $M(\theta) = \sum_{\alpha_1,\ldots,\alpha_N \in \{0,1\}} M_{\alpha_1,\ldots,\alpha_N}\theta_1^{\alpha_1}\cdots\theta_N^{\alpha_N}$, where $\alpha_1,\ldots,\alpha_N \in \{0,1\}$ and $M_{\alpha_1,\ldots,\alpha_N}$ denotes the coefficient matrix.

Definition 2. (DMAR problem): Given an RPM $G(\theta) \in \mathbb{R}^{p\times q}(\theta)$, find matrices $M_1(\theta)$, $M_2(\theta)$, and $M_3(\theta)$ such that

$$G(\theta) = M_1(\theta)M_2^{-1}(\theta)M_3(\theta) \tag{3}$$

where $M_1(\theta) \in \mathbb{R}^{p\times r}[\theta]$, $M_2(\theta) \in \mathbb{R}^{r\times r}[\theta]$, and $M_3(\theta) \in \mathbb{R}^{r\times q}[\theta]$ are multiaffine in θ, and r denotes the dimension of the DMAR. We also call $(M_1(\theta), M_2(\theta), M_3(\theta), r)$ a DMAR of RPM $G(\theta)$.

Since $M_1(\theta)$, $M_2(\theta)$, and $M_3(\theta)$ are multiaffine in θ, they can be expressed in the form of

$$M_i(\theta) = \sum_{\alpha_1,\ldots,\alpha_N \in \{0,1\}} \theta_1^{\alpha_1}\cdots\theta_N^{\alpha_N} M^{(i)}_{\alpha_1,\ldots,\alpha_N} \tag{4}$$

where $M^{(i)}_{\alpha_1,\ldots,\alpha_N}$ is the coefficient matrix. Moreover, we also use $\left(M^{(1)}_{\alpha_1,\ldots,\alpha_N},\ M^{(2)}_{\alpha_1,\ldots,\alpha_N},\ M^{(3)}_{\alpha_1,\ldots,\alpha_N},\ r\right)$ to represent the DMAR $(M_1(\theta), M_2(\theta), M_3(\theta), r)$.

Remark 1: DMAR, as a new type of uncertainty system modeling, happens to be very suitable for deriving S-variable LMI results [10] and can reduce the numerical burden [14]. As the LFR is a special structure of DMAR [10], the existing LFR modeling methods, e.g., the object-oriented LFR procedure [15] and the elementary operation approach [16] can be used to build trivially a DMAR. However, as pointed by Peaucelle et al. [10], the LFR-based methods often generate high-dimensional DMAR, and the equivalent DMAR models may lead to robust control with more conservatism or less conservatism. Therefore, in order to obtain a robust control with less conservatism, a feasible method is given to first reduce the dimension of the given higher dimensional DMAR and then choose the better one with less conservatism.

B. Constrained CIS

Definition 3 (See [7]): A subspace $\mathcal{W}$ is called a right CIS of $M_1,\ldots,M_n \in \mathbb{R}^{r\times r}$ if $M_i\boldsymbol{w} \in \mathcal{W}, i = 1,\ldots,n$, for all $\boldsymbol{w} \in \mathcal{W}$, and a right CCIS if, in addition, $C\boldsymbol{w} = \boldsymbol{0}$ for arbitrarily given $C \in \mathbb{R}^{p\times r}$. Dually, a subspace $\mathcal{W}$ is called a left CIS of $M_1,\ldots,M_n$ if $\boldsymbol{w}^\top M_i \in \mathcal{W}$, $i = 1,\ldots,n$, for all $\boldsymbol{w} \in \mathcal{W}$, and a left CCIS if, in addition, $\boldsymbol{w}^\top C = \boldsymbol{0}$ for arbitrarily given $C \in \mathbb{R}^{r\times q}$. A right/left CCIS $\mathcal{W}$ is called trivial if $\dim(\mathcal{W}) = 0$; otherwise, it is nontrivial.

Definition 4 (See [7]): Given a matrix $F \in \mathbb{R}^{m\times r}$ and a matrix set $\mathcal{M} = \{M_1,\ldots,M_n\}$ with $M_i \in \mathbb{R}^{r\times r}$, the matrix function $\mathcal{L}(\mathcal{M}, F)$ is defined as

$$\mathcal{L}(\mathcal{M}, F) := \left[(FM_1)^\top \quad \cdots \quad (FM_n)^\top\right]^\top \in \mathbb{R}^{nm\times r}. \tag{5}$$

Let $C \in \mathbb{R}^{p\times r}$ and then the matrix functions $\mathcal{L}_k(\mathcal{M}, C)$ and $\mathcal{F}_k(\mathcal{M}, C)$ are recursively defined by

$$\mathcal{L}_k(\mathcal{M}, C) := \mathcal{L}(\mathcal{M}, \mathcal{L}_{k-1}(\mathcal{M}, C)) \in \mathbb{R}^{pn^{k-1}\times r} \tag{6a}$$

$$\mathcal{F}_k(\mathcal{M}, C) := \begin{bmatrix} \mathcal{L}_1(\mathcal{M}, C) \\ \vdots \\ \mathcal{L}_k(\mathcal{M}, C) \end{bmatrix} \in \mathbb{R}^{\frac{p\left(1-n^k\right)}{1-n}\times r} \tag{6b}$$

with $\mathcal{F}_1(\mathcal{M}, C) = \mathcal{L}_1(\mathcal{M}, C) = C$ and $k = 2,\ldots,r$.

Lemma 1 (See [7]): There exists a nontrivial right CCIS $\mathcal{W}$ of $M_1,\ldots,M_n \in \mathbb{R}^{r\times r}$ constrained by $C \in \mathbb{R}^{p\times r}$ if and only if $\mathcal{F}_r(\mathcal{M}, C)$ is rank deficient. Moreover, $\mathcal{W} = \ker(\mathcal{F}_r(\mathcal{M}, C))$.

Remark 2: Lemma 1 necessitates the computation of the matrix $\mathcal{F}_r(\mathcal{M}, C)$ of size $\frac{p(1-n^r)}{1-n} \times r$ and its kernel. A commonly employed method for kernel computation is singular value decomposition with computational complexity of $O(\frac{p(1-n^r)}{1-n} \times r^2)$ [17]. Consequently, both the memory storage requirement for $\mathcal{F}_r(\mathcal{M}, C)$ and the computation time to solve its kernel will increase dramatically as n and r increase. Furthermore, when dealing with large size matrices in Gaussian elimination, the number of arithmetic operations, such as additions, multiplications, and divisions, becomes substantial. This not only increases computational burden but also introduces potential inaccuracies due to floating-point limitations, making kernel computation even more challenging. Even with relatively small values, such as $n = r = p = 10$, a very huge matrix $\mathcal{F}_r(\mathcal{M}, C)$ needs to be calculated, which requires approximately $\frac{10(10^{10}-1)}{9} \times 10 \approx 1.11 \times 10^{11}$ memory storage and $O(1.11 \times 10^{12})$ complexity for solving the kernel of $\mathcal{F}_r(\mathcal{M}, C)$, further emphasizing the computational challenges.

Definition 5: For a DMAR $(M_1(\theta), M_2(\theta), M_3(\theta), r)$, a subspace $\mathcal{W}$ is said to be a right CCIS of $(M_1(\theta), M_2(\theta), M_3(\theta), r)$ if it is a right CCIS of $\{M^{(2)}_{\alpha_1,\ldots,\alpha_n}\}$ constrained by

$$M^{(1)} = \begin{bmatrix} M^{(1)}_{0,\ldots,0} \\ \vdots \\ M^{(1)}_{1,\ldots,1} \end{bmatrix} \tag{7}$$

where $M^{(1)}_{\alpha_1,\ldots,\alpha_n}$ and $M^{(2)}_{\alpha_1,\ldots,\alpha_n}$ are defined in (4), $\alpha_1,\ldots,\alpha_N \in \{0,1\}$. Dually, a subspace $\mathcal{W}$ is said to be a left CCIS of $(M_1(\theta), M_2(\theta), M_3(\theta), r)$ if it is a left CCIS of $\{M^{(2)}_{\alpha_1,\ldots,\alpha_n}\}$ constrained by

$$M^{(3)} = \left[M^{(3)}_{0,\ldots,0,0} \quad M^{(3)}_{0,\ldots,0,1} \quad \cdots \quad M^{(3)}_{1,\ldots,1,0} \quad M^{(3)}_{1,\ldots,1,1}\right] \tag{8}$$

where $M^{(2)}_{\alpha_1,\ldots,\alpha_n}$ and $M^{(3)}_{\alpha_1,\ldots,\alpha_n}$ are defined in (4), $\alpha_1,\ldots,\alpha_N \in \{0,1\}$.

Remark 3: It should be noted that the number of coefficient matrices $M^{(2)}_{\alpha_1,\ldots,\alpha_N}$ in matrices $M_2(\theta)$ is $n = 2^N$, which grows exponentially with the number of parameters N. Moreover, it follows from Remark 1 that the dimension r of the DMAR, i.e., the size of the coefficient

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matrices of the matrix $M_2(\theta)$ is usually very large. However, the method in [7] requires the computation of the matrix $\mathcal{F}_r(\mathcal{M}, C)$ with dimension $\frac{p(1-n^r)}{1-n} \times r$, which increases dramatically as n and r increase. Thus, in order to apply CCIS to DMAR, it is necessary to develop efficient CCIS computation methods.

III. Efficient CCIS to DMAR of Rational Parameter Systems

This section presents new necessary and sufficient conditions for computing CCIS, based on which reducibility conditions for DMAR are given.

A. Efficient Constrained Computation of CCIS

Lemma 2: Let $H \in \mathbb{R}^{m_h \times r}$ and $V \in \mathbb{R}^{m_v \times r}$ and define

$$F := \begin{bmatrix} H \\ V \end{bmatrix}. \tag{9}$$

Consider a set of matrices $\mathcal{M} = \{M_1, \ldots, M_n\}$ with $M_i \in \mathbb{R}^{r \times r}$. If the matrix V can be obtained from the matrix H by a series of elementary row transformations $T \in \mathbb{R}^{m_v \times m_h}$, i.e.,

$$V = TH \tag{10}$$

then $\text{span}_{\text{row}}(\mathcal{L}(\mathcal{M}, F)) = \text{span}_{\text{row}}(\mathcal{L}(\mathcal{M}, H))$.

Proof: It follows from (10) that:

$$\text{span}_{\text{row}}(VM_i) = \text{span}_{\text{row}}(THM_i) \subseteq \text{span}_{\text{row}}(HM_i). \tag{11}$$

In view of (9), (11), and Definition 4, we derive that

$$\begin{aligned} &\text{span}_{\text{row}}(\mathcal{L}(\mathcal{M}, F)) = \text{span}_{\text{row}}(FM_1) + \cdots + \text{span}_{\text{row}}(FM_n) \\ &= \text{span}_{\text{row}}\left(\begin{bmatrix} HM_1 \\ VM_1 \end{bmatrix}\right) + \cdots + \text{span}_{\text{row}}\left(\begin{bmatrix} HM_n \\ VM_n \end{bmatrix}\right) \\ &= \text{span}_{\text{row}}(HM_1) + \cdots + \text{span}_{\text{row}}(HM_n) \\ &= \text{span}_{\text{row}}(\mathcal{L}(\mathcal{M}, H)). \end{aligned} \tag{12}$$

■

Theorem 1: Define H_k as the matrix whose row vectors form a basis of the rows of $L_k = \mathcal{L}(\mathcal{M}, H_{k-1})$, with $L_1 = C$ and $k = 2, \ldots, r$. Let

$$F_k(\mathcal{M}, C) := \begin{bmatrix} H_1^\top & \cdots & H_k^\top \end{bmatrix}^\top. \tag{13}$$

Then, there exists a nontrivial right CCIS $\mathcal{W}$ of $M_1, \ldots, M_n \in \mathbb{R}^{r \times r}$ constrained by $C \in \mathbb{R}^{p \times r}$ if and only if $F_r(\mathcal{M}, C)$ is rank deficient. Moreover, $\mathcal{W} = \ker(F_r(\mathcal{M}, C))$.

Proof: Since the row vectors of H_k are the basis vectors of the rows of the matrix $L_k = \mathcal{L}(\mathcal{M}, H_{k-1})$, we can divide the row vectors of the matrix L_k into two parts, i.e., H_k consisting of the basis vectors and the matrix V_k consisting of the remaining row vectors, which can be expressed as $V_k = T_k H_k$ for some matrix T_k. Leveraging Lemma 2, we find that $\text{span}_{\text{row}}(\mathcal{L}(\mathcal{M}, L_k)) = \text{span}_{\text{row}}(\mathcal{L}(\mathcal{M}, H_k))$. By the definitions of $F_k(\mathcal{M}, C)$ of (13) and $\mathcal{F}_k(\mathcal{M}, C)$ of (6b), $\text{span}_{\text{row}}(F_k(\mathcal{M}, C)) = \text{span}_{\text{row}}(\mathcal{F}_k(\mathcal{M}, C))$. By Lemma 1, there exists a nontrivial right CCIS $\mathcal{W}$ of $M_1, \ldots, M_n \in \mathbb{R}^{r \times r}$ constrained by $C \in \mathbb{R}^{p \times r}$ if and only if $F_r(\mathcal{M}, C)$ is rank deficient. ■

Theorem 1 represents an efficient algorithm to derive a right CCIS of $M_1, \ldots, M_n \in \mathbb{R}^{r \times r}$ constrained by $C \in \mathbb{R}^{p \times r}$, which is described by the pseudocode in Algorithm 1.

Given the definitions of matrices H_k and L_k in Theorem 1, it is established that both matrices comprise p columns. Furthermore, the row vectors of H_k serve as a set of basis vectors for L_k, suggesting that the dimensions of H_k are constrained to be no greater than $p \times p$. This construction directly restricts the dimensions of the matrix $F_r(\mathcal{M}, C)$ specified in (13), so that the size of $F_r(\mathcal{M}, C)$ does not exceed $rp \times p$. This condition emphasizes that the dimension of $F_r(\mathcal{M}, C)$ grows linearly with r, independent of the number of matrices, n. In a particular instance where $n = r = p = 10$, the dimensions of $F_r(\mathcal{M}, C)$ are thus confined to be less than 10^3. This result significantly reduces memory requirements, taking only $\frac{9}{10^9}$ of the memory compared with the previous method [7].

Algorithm 1: Efficient Computation of a Right CCIS.

Input: $M_1, \ldots, M_n \in \mathbb{R}^{r \times r}$, $C \in \mathbb{R}^{p \times r}$.
Output: $\mathcal{W}$.
Initialize H_1 so that its row vectors form a basis for the rows of $L_1 = C$;
for $k = 2$ **to** r **do**
  Compute $L_k \leftarrow \mathcal{L}(\mathcal{M}, H_{k-1})$;
  Determine H_k such that its row vectors form a basis for the rows of L_k;
Construct $F_r(\mathcal{M}, C)$ as in (13);
Compute $\mathcal{W} \leftarrow \ker(F_r(\mathcal{M}, C))$.

Example 1: To demonstrate the specifics and effectiveness of Algorithm 1, consider the following matrices:

$$M_1 = \begin{bmatrix} 1 & -1 & -1 & 1 \\ -2 & 4 & 2 & 0 \\ 6 & -6 & -2 & -1 \\ 0 & -3 & -2 & 3 \end{bmatrix}, \; M_2 = \begin{bmatrix} 15 & -4 & -1 & -5 \\ -15 & 8 & 1 & 6 \\ 26 & -10 & -2 & -9 \\ 40 & -13 & -2 & -14 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} -6 & 4 & 1 & 2 \\ 10 & -9 & -3 & -3 \\ -7 & 7 & 1 & 2 \\ -18 & 14 & 3 & 6 \end{bmatrix}, \; M_4 = \begin{bmatrix} 9 & -8 & -2 & -4 \\ -19 & 17 & 3 & 9 \\ 22 & -0 & -1 & -12 \\ 32 & -30 & -6 & -15 \end{bmatrix},$$

$$C^\top = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}. \tag{14}$$

By means of Algorithm 1, we obtain $F_4(\mathcal{M}, C) = [H_1^\top \;\; H_2^\top \;\; H_3^\top \;\; H_4^\top]^\top$ with $H_1 = C$ and

$$\begin{aligned} H_2 &= \begin{bmatrix} 7 & -4 & -1 & -3 \\ 1 & -1 & -1 & 0 \end{bmatrix}, \; H_3 = \begin{bmatrix} 9 & -8 & -7 & -1 \\ -3 & 1 & -1 & 2 \end{bmatrix} \\ H_4 &= \begin{bmatrix} -17 & 4 & -9 & 13 \\ -11 & 7 & 3 & 4 \end{bmatrix}. \end{aligned} \tag{15}$$

The kernel of $F_4(\mathcal{M}, C)$ is $\mathcal{W} = \text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ with

$$\boldsymbol{v}_1 = \begin{bmatrix} -1 & -2 & 1 & 0 \end{bmatrix}^\top, \; \boldsymbol{v}_2 = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}^\top \tag{16}$$

which is a right CCIS of $\mathcal{M}$ constrained by C.

Remark 4: To compute the CCIS, the method given in [7] requires the matrix $\mathcal{F}_r(\mathcal{M}, C)$ with dimension $\frac{p(1-n^r)}{1-n} \times r = \frac{1(1-4^4)}{1-4} \times 4 = 85 \times 4$. In contrast, the size of the matrix $F_r(\mathcal{M}, C)$ required in the proposed CCIS efficient calculation method is 7×4, the memory storage of which is significantly less than that of the method given in [7].

B. Reducibility of DMAR With CCIS

Definition 6. (Reducibility of DMARs): For a given DMAR $(M_1(\theta), M_2(\theta), M_3(\theta), r)$, if there is a new DMAR

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$(\hat{M}_1(\theta), \hat{M}_2(\theta), \hat{M}_3(\theta), \hat{r})$ such that

$$M_1(\theta)M_2^{-1}(\theta)M_3(\theta) = \hat{M}_1(\theta)\hat{M}_2^{-1}(\theta)\hat{M}_3(\theta),\ \hat{r} < r \tag{17}$$

then the given DMAR $(M_1(\theta), M_2(\theta), M_3(\theta), r)$ is reducible.

Now, sufficient conditions for the reducibility of DMARs can be given by utilizing right or left CCIS as follows.

Theorem 2: If a DMAR $(M_1(\theta), M_2(\theta), M_3(\theta), r)$ admits a nontrivial right CCIS $\mathcal{W}$ with a basis $\{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{\tilde{r}}\}$, then it is reducible. Moreover, a low-dimensional DMAR $(\hat{M}_1(\theta),\ \hat{M}_2(\theta), \hat{M}_3(\theta), \hat{r}) = (M_1(\theta)\hat{R}, \hat{L}M_2(\theta)\hat{R}, \hat{L}M_3(\theta), r)$ can be obtained such that (17) holds true, where $\hat{r} = r - \tilde{r} < r$ and

$$R := \begin{bmatrix}\boldsymbol{w}_1 & \cdots & \boldsymbol{w}_{\tilde{r}} \mid \boldsymbol{w}_{\tilde{r}+1} & \cdots & \boldsymbol{w}_r\end{bmatrix} := \begin{bmatrix}\tilde{R} \mid \hat{R}\end{bmatrix} \tag{18a}$$

$$L := \begin{bmatrix}\tilde{L}^\top & \hat{L}^\top\end{bmatrix}^\top := R^{-1} \tag{18b}$$

with $\tilde{L} \in \mathbb{R}^{\tilde{r}\times r}$, $\hat{L} \in \mathbb{R}^{\hat{r}\times r}$, and $\boldsymbol{w}_{\tilde{r}+1} \cdots \boldsymbol{w}_r$ being selected such that R is a nonsingular matrix.

Proof: Following the proof of Theorem 6 in [7], we derive:

$$\begin{aligned}
&M_1(\theta)M_2^{-1}(\theta)M_3(\theta) = M_1(\theta)RR^{-1}M_2^{-1}(\theta)L^{-1}LM_3(\theta)\\
&= M_1(\theta)R(LM_2(\theta)R)^{-1}LM_3(\theta)\\
&= \begin{bmatrix}\mathbf{0} & M_1(\theta)\hat{R}\end{bmatrix}\begin{bmatrix}* & *\\ \mathbf{0} & \hat{L}M_2(\theta)\hat{R}\end{bmatrix}^{-1}\begin{bmatrix}*\\ \hat{L}M_3(\theta)\end{bmatrix}\\
&= \left(M_1(\theta)\hat{R}\right)\left(\hat{L}M_2(\theta)\hat{R}\right)^{-1}\left(\hat{L}M_3(\theta)\right)\\
&= \hat{M}_1(\theta)\hat{M}_2^{-1}(\theta)\hat{M}_3(\theta)
\end{aligned}$$

where $*$ denotes some appropriate matrices. ■

Theorem 3: If a DMAR $(M_1(\theta), M_2(\theta), M_3(\theta), r)$ admits a nontrivial left CCIS $\mathcal{W}$ with a basis $\{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{\tilde{r}}\}$, then it is reducible. Moreover, a low-dimensional DMAR $(\hat{M}_1(\theta),\ \hat{M}_2(\theta), \hat{M}_3(\theta), \hat{r})$= $(M_1(\theta)\hat{R}, \hat{L}M_2(\theta)\hat{R}, \hat{L}M_3(\theta), r)$ can be obtained such that (17) holds true, where $\hat{r} = r - \tilde{r} < r$ and

$$L^\top := \begin{bmatrix}\boldsymbol{w}_1 & \cdots & \boldsymbol{w}_{\tilde{r}} \mid \boldsymbol{w}_{\tilde{r}+1} & \cdots & \boldsymbol{w}_r\end{bmatrix} := \begin{bmatrix}\tilde{L}^\top \mid \hat{L}^\top\end{bmatrix} \tag{19a}$$

$$R := \begin{bmatrix}\tilde{R} \mid \hat{R}\end{bmatrix} := L^{-1} \tag{19b}$$

with $\tilde{R} \in \mathbb{R}^{r\times\tilde{r}}$, $\hat{R} \in \mathbb{R}^{r\times\hat{r}}$, and $\boldsymbol{w}_{\tilde{r}+1} \cdots \boldsymbol{w}_r$ being selected such that L is a nonsingular matrix.

Since Theorem 3 can be proved similarly to Theorem 2, the details are omitted here for the sake of brevity.

Example 2: Consider the following DMAR: with dimension $r = 12$. By Algorithm 1, we can obtain a 7-D right CCIS of $(M_1(\theta), M_2(\theta), M_3(\theta), r)$. Thus, the DMAR of (20), shown at the bottom of this page, can be exactly reduced. By Theorem 2, we can obtain a 5-D DMAR with

$$\hat{M}_1(\theta)=\begin{bmatrix}\theta_1 & 0 & 0 & \theta_1 & 1\\ 2 & \theta_3 & 0 & 1 & 0\\ 1 & 0 & \theta_1\theta_3 & 0 & 0\\ \theta_1 & 0 & 0 & 0 & 0\\ 0 & \theta_2\theta_3 & 0 & \theta_3 & 1\end{bmatrix}, \hat{M}_3(\theta)=\begin{bmatrix}2 & 0 & 0 & -1 & 0\\ 1 & 1 & 0 & 2 & 0\\ 0 & 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 0 & 0\\ -1 & 1 & 0 & 0 & 1\end{bmatrix}$$

$$\hat{M}_2(\theta) = \begin{bmatrix}\theta_1 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 0\\ \theta_3 & 1 & \theta_1\,\theta_2 & 0 & 0\\ \theta_2 & 0 & 1 & \theta_1\,\theta_3 & \theta_2\,\theta_3\\ 0 & -8 & \theta_1\,\theta_2\,\theta_3 & 0 & 1\end{bmatrix}. \tag{21}$$

Remark 5: It should be noted that in the computation of a right CCIS of $(M_1(\theta), M_2(\theta), M_3(\theta), r)$, the method given in [7] requires the matrix $\mathcal{F}_r(\mathcal{M}, C)$ with dimension $\frac{40(1-8^{12})}{1-8} \times 12 \approx 4.71 \times 10^{12}$. However, the size of the matrix $F_r(\mathcal{M}, C)$ required in this study is $5 \times 12 \times 12 = 720$ and its memory storage is much smaller than that of [7].

IV. H_2 Control Designs With CCIS

In this section, we begin with proposing an H_2 feedback controller synthesis method for rational parameter systems based on the DMAR model. Experimental results demonstrate that the proposed H_2 feedback controller synthesis method is less conservative compared with existing methods. Furthermore, the section underscores the advantages of employing the efficient CCIS method in the design of H_2 feedback

$$M_1(\theta)=\begin{bmatrix}\theta_1 & 0 & 0 & \theta_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 2 & \theta_3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & 0 & \theta_1\theta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \theta_2\theta_3 & 0 & \theta_3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$M_2(\theta)=\begin{bmatrix}\theta_1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \theta_3 & 1 & \theta_1\,\theta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \theta_2 & 0 & 1 & \theta_1\,\theta_3 & \theta_2\,\theta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -8 & \theta_1\,\theta_2\,\theta_3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 1 & 1 & \theta_1\,\theta_3 & 0 & 0 & 1 & \theta_2 & 0 & 1 & 0 & \theta_2\,\theta_3\\ 1 & 0 & 0 & 0 & \theta_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & \theta_1\,\theta_2\,\theta_3 & 0 & 1 & 0 & 0\\ 0 & \theta_1\,\theta_2 & 1 & 1 & 0 & 0 & \theta_1\,\theta_3 & 1 & 0 & 0 & 1 & 0\\ 1 & 0 & 1 & 2 & 0 & 1 & \theta_3 & 0 & 0 & 1 & \theta_2 & 1\\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & \theta_2 & 1 & 1 & 0 & 0\end{bmatrix}$$

$$M_3(\theta)=\begin{bmatrix}2 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 2\\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & -1\\ -1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1\\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0\end{bmatrix}^\top \tag{20}$$

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controllers, highlighting its superiority on enhancing controller performance. To do this, we first give some notation preparations.

Notation 1: Let θ and θ_+ denote the value of parameters $\theta = (\theta_1, \ldots, \theta_N) \in \Theta$ at time k and $k+1$, respectively. The set Θ is defined as the Cartesian product of Θ_i, which represents the convex hull of the vertices $\theta_i^{[1]}, \ldots, \theta_i^{[v_i]}$, i.e., $\theta_i \in \Theta_i$. Each parameter $\theta_i \in \mathbb{R}^{m_i}$ can be written as the weighted sum of the v_i vertices $\theta_i^{[1]}, \ldots, \theta_i^{[v_i]}$ of Θ_i, i.e., $\theta_i = \sum_{l_i=1}^{v_i} \alpha_{i,l_i}\theta_i^{[l_i]}$ with $\sum_{l_i=1}^{v_i} \alpha_{i,l_i} = 1$ and $\alpha_{i,l_i} \geq 0$, $l_i = 1, \ldots, v_i$. We denote by $\mathcal{V} = \mathcal{V}_1 \times \cdots \times \mathcal{V}_n$ the finite set of all extremal values of the parameters gathered in θ. A generic element of $\mathcal{V}$ will be denoted by $\theta^{[\mathbf{v}]}$ with $\mathbf{v} = (v_1, \ldots, v_n)$ being the vector of indices of vertices for each component. $\mathcal{I}$ is the set of all vectors of indices $\mathbf{v}$. $\theta^{[\mathbf{v}]}$ is the one-to-one map from $\mathcal{I}$ to $\mathcal{V}$. The total number of vertices in Θ is $N_\theta = \prod_{i=1}^{N} v_i$. $\mathrm{Tr}(A)$ denotes the trace of the matrix A. $A \succ B$ and $A \prec B$ mean that $A - B$ is symmetric positive definite and negative definite, respectively.

A. H_2 Control Designs for Discrete-Time LPV Systems

1) H_2 State-Feedback Based on DMAR: Consider the following discrete-time rational parameter system (DTRPS):

$$x_{k+1} = A_{xx}(\theta)x_k + B_{xu}(\theta)u_k + B_{xw}(\theta)w_k \tag{22a}$$

$$z_k = C_{zx}(\theta)x_k + D_{zu}(\theta)u_k \tag{22b}$$

where $w_k \in \mathbb{R}^{m_w}$ represents the exogenous input vector and $B_{xw}(\theta)$ is multiaffine, and the other matrices are rational with respect to time-varying parameters θ. Then, its H_2 performance [18] is defined by $\|T_{zw}\|_2^2 := \lim_{T\to\infty} \sup_{\theta\in\Theta} \mathcal{E}\{\frac{1}{T}\sum_{k=0}^{T} z_k^\top z_k\}$, where the system input w_k is a zero-mean white noise Gaussian process with identity covariance matrix and $\mathcal{E}$ denotes the mathematical expectation. Suppose a DMAR of (22) is given by

$$\begin{aligned}\begin{bmatrix} A_{xx}(\theta) & B_{xu}(\theta) \\ C_{zx}(\theta) & D_{zu}(\theta)\end{bmatrix} &= \begin{bmatrix} E_x(\theta) \\ E_z(\theta)\end{bmatrix} E_\pi^{-1}(\theta)M_3(\theta) \\ &= M_1(\theta)M_2^{-1}(\theta)M_3(\theta)\end{aligned} \tag{23}$$

with $M_3(\theta) = [A(\theta) \quad B(\theta)]$. The transposed version of (23) is given by

$$\begin{bmatrix} A_{xx}^\top(\theta) & C_{zx}^\top(\theta) \\ B_{xu}^\top(\theta) & D_{zu}^\top(\theta)\end{bmatrix} = \begin{bmatrix} E_{dx}(\theta) \\ E_{dy}(\theta)\end{bmatrix} E_{d\pi}^{-1}(\theta)M_{3d}(\theta) \tag{24}$$

with $M_{3d}(\theta) = [A_d(\theta) \quad B_d(\theta)]$.

Let $u_k = Kx_k$, and then the closed-loop system of (22) is

$$x_{k+1} = A_{xx,K}(\theta)x_k + B_{xw}(\theta)w_k \tag{25a}$$

$$z_k = C_{zx,K}(\theta)x_k \tag{25b}$$

with $A_{xx,K}(\theta) = A_{xx}(\theta) + B_{xu}(\theta)K$ and $C_{zx,K}(\theta) = C_{zx}(\theta) + D_{zu}(\theta)K$. Its H_2 performance is equal to the performance of the dual system defined by

$$x_{d,k+1} = A_{xx,K}^\top(\theta)x_{d,k} + C_{zx,K}^\top(\theta)w_{d,k} \tag{26a}$$

$$z_{d,k} = B_{xw}^\top(\theta)x_{d,k}. \tag{26b}$$

According to (24), (26a) can be rewritten as

$$\begin{aligned} x_{d,k+1} &= (E_{dx}(\theta) + K^\top E_{dy}(\theta)) \\ &\quad \times E_{d\pi}^{-1}(\theta)(A_d(\theta)x_{d,k} + B_d(\theta)w_{d,k}) \\ &= (E_{dx}(\theta) + K^\top E_{dy}(\theta))(-\pi_{d,k})\end{aligned} \tag{27}$$

with

$$\pi_{d,k} := -E_{d\pi}^{-1}(\theta)(A_d(\theta)x_{d,k} + B_d(\theta)w_{d,k}). \tag{28}$$

Define

$$\eta_{d,k}^\top := \begin{bmatrix} x_{d,k+1}^\top & \pi_{d,k}^\top & x_{d,k}^\top & w_{d,k}^\top\end{bmatrix} \tag{29a}$$

$$E_d(\theta) := \begin{bmatrix} I & E_{dx}(\theta) + K^\top E_{dy}(\theta) & 0 & 0 \\ 0 & E_{d\pi}(\theta) & A_d(\theta) & B_d(\theta)\end{bmatrix}. \tag{29b}$$

Then, it follows from (27)–(29) that $E_d(\theta)\eta_{d,k} = 0$.

Let

$$\begin{aligned} N_{dx}(\theta) &= \begin{bmatrix} S_{dx} & S_{dx}E_{dx}(\theta) + S_{dy}E_{dy}(\theta) & 0 & 0\end{bmatrix} \\ N_{d\pi}(\theta) &= \begin{bmatrix} 0 & E_{d\pi}(\theta) & A_d(\theta) & B_d(\theta)\end{bmatrix}\end{aligned} \tag{30}$$

with $S_{dy} = S_{dx}K^\top$. Then, the H_2 state-feedback controller for the DTRPS can be given by the following result.

Theorem 4: If there exist N_θ symmetric positive-definite matrices (SPDMs) $P_d(\theta^{[l]})$, $P_d(\theta_+^{[l]})$, and $S(\theta^{[l]})$, and matrices S_{dx}, S_{dy}, and $S_{d\pi}$ of appropriate size satisfying the following LMIs simultaneously:

$$\begin{aligned} &\begin{bmatrix} S(\theta^{[l]}) & B_{xw}^\top(\theta^{[l]}) \\ B_{xw}(\theta^{[l]}) & P_d(\theta^{[l]})\end{bmatrix} \succ 0 \\ &\mathrm{diag}\left\{P_d(\theta_+^{[l]}),\ 0,\ -P_d(\theta^{[l]}),\ -I\right\} \\ &\prec \left\{\begin{bmatrix} I \\ 0\end{bmatrix} N_{dx}(\theta^{[l]})\right\}^s + \left\{S_{d\pi}N_{d\pi}(\theta^{[l]})\right\}^s\end{aligned} \tag{31}$$

then the state-feedback gain $K = S_{dy}^\top(S_{dx}^\top)^{-1}$ guarantees that the closed-loop has an H_2 upper bound

$$\|T_{zw}\|_2 \leq \min_{P_d(\theta^{[l]})} \max_{S(\theta^{[l]})} \left(\sqrt{\mathrm{Tr}(S(\theta^{[l]}))}\right) \tag{32}$$

whatever $\theta \in \Theta$.

Proof: Thanks to the convexity of matrix inequalities that the LMIs (31) hold if and only if for all uncertainties $\theta \in \Theta$, one has

$$\begin{aligned} &\mathrm{diag}\left\{P_d(\theta_+),\ 0,\ -P_d(\theta),\ -I\right\} \\ &\prec \left\{\begin{bmatrix} I \\ 0\end{bmatrix} N_{dx}(\theta)\right\}^s + \left\{S_{d\pi}N_{d\pi}(\theta)\right\}^s.\end{aligned} \tag{33}$$

With the change of variable $S_{dy} = S_{dx}K^\top$, (33) reads as

$$\mathrm{diag}\left\{P_d(\theta_+),\ 0,\ -P_d(\theta),\ -I\right\} \prec \left\{S_dE_d(\theta)\right\}^s \tag{34}$$

with $S_d = \left[\begin{array}{c|c} S_{dx} & S_{d\pi} \\ 0 & \end{array}\right]$. After congruence operation of $\eta_{d,k} \neq 0$ on this last matrix inequality, one gets along the trajectories ($E_d(\theta)\eta_{d,k} = 0$)

$$x_{d,k+1}^\top P_d(\theta_+)x_{d,k+1} - x_{d,k}^\top P_d(\theta)x_{d,k} - w_{d,k}^\top w_{d,k} < 0.$$

By Schur complement theorem [19], we derive from $\begin{bmatrix} S(\theta) & B_{xw}^\top(\theta) \\ B_{xw}(\theta) & P_d(\theta)\end{bmatrix} \succ 0$ that $\mathrm{Tr}(B_{xw}^\top(\theta)P_d^{-1}(\theta)B_{xw}(\theta)) < \mathrm{Tr}(S(\theta))$. Then, taking into account the discrete-time version of the generalized H_2 norm condition provided by Scherer et al. [20] that $\|T_{zw}\|_2^2 < \mathrm{Tr}(B_{xw}^\top(\theta)P_d^{-1}(\theta)B_{xw}(\theta))$, one can obtain (32). ■

2) H_2 Gain-Scheduled State-Feedback Control Based on DMAR: Consider the DMAR of (23) with M_3 being a constant matrix without containing parameter θ. So, we can rewrite M_3 as $M_3 = [A \quad B]$, and the transposed version of (3) becomes

$$\begin{bmatrix} A_{xx}^\top(\theta) & C_{zx}^\top(\theta) \\ B_{xu}^\top(\theta) & D_{zu}^\top(\theta)\end{bmatrix} = \begin{bmatrix} E_{dx} \\ E_{dy}\end{bmatrix} E_{d\pi}^{-1}(\theta)M_{3d}(\theta) \tag{35}$$

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with $M_{3d}(\theta) = [A_d(\theta) \quad B_d(\theta)]$.

Remark 6: It should be noted that the above-mentioned special DMAR form can be derived from LFR, which is to express a rational matrix $G(\theta)$ as $G(\theta) = M_{11} + M_{12}\Delta(\theta)(I - M_{22}\Delta(\theta))^{-1}M_{21}$, with $\Delta(\theta)$ being linear in the elements of θ [21]. Then, the LFR can be converted into a DMAR in the form of (3). If $M_{11} = \mathbf{0}$, then we have $M_1(\theta) = M_{12}\Delta(\theta)$, $M_2(\theta) = I - M_{22}\Delta(\theta)$, and $M_3(\theta) = M_{21}$; otherwise, we have

$$M_1(\theta) = \begin{bmatrix} M_{11} & M_{12}\Delta(\theta) \end{bmatrix}$$
$$M_2(\theta) = \begin{bmatrix} I & 0 \\ 0 & I - M_{22}\Delta(\theta) \end{bmatrix}, \; M_3(\theta) = \begin{bmatrix} I \\ M_{21} \end{bmatrix}.$$

Then, considering gain-scheduled control $K(\theta)$, we can derive the following results for system (26) $E_{dn}(\theta)\eta_{d,k} = 0$, where

$$\eta_{d,k}^\top = \begin{bmatrix} x_{d,k+1}^\top & \pi_{d,k}^\top & x_{d,k}^\top & w_{d,k}^\top \end{bmatrix}$$
$$E_{dn}(\theta) = \begin{bmatrix} I & E_{dx}(\theta) + K^\top(\theta)E_{dy}(\theta) & 0 & 0 \\ 0 & E_{d\pi}(\theta) & A_d(\theta) & B_d(\theta) \end{bmatrix}.$$

Define

$$\eta_{d,k}^\top := \begin{bmatrix} x_{d,k+1}^\top & \pi_{d,k}^\top & x_{d,k}^\top & w_{d,k}^\top \end{bmatrix} \tag{36a}$$
$$E_{dg}(\theta) := \begin{bmatrix} I & E_{dx} + K^\top(\theta)E_{dy} & 0 & 0 \\ 0 & E_{d\pi}(\theta) & A_d(\theta) & B_d(\theta) \end{bmatrix}. \tag{36b}$$

Then, it follows from (36) that $E_{dg}(\theta)\eta_{d,k} = 0$.

Let

$$N_{dgx}(\theta) = \begin{bmatrix} S_{dx}(\theta) & S_{dx}(\theta)E_{dx} + S_{dy}(\theta)E_{dy} & 0 & 0 \end{bmatrix} \tag{37a}$$
$$N_{dg\pi}(\theta) = \begin{bmatrix} 0 & E_{d\pi}(\theta) & A_d(\theta) & B_d(\theta) \end{bmatrix} \tag{37b}$$

with $S_{dy}(\theta) = S_{dx}(\theta)K^\top(\theta)$. Then, the H_2 gain-scheduled state-feedback controller for the DTRPS can be given by the following result.

Theorem 5: If there exist N_θ SPDMs $P_d(\theta^{[l]})$ and $P_d(\theta_+^{[l]})$ and matrices $S(\theta^{[l]})$, $S_{dx}(\theta^{[l]})$, $S_{dy}(\theta^{[l]})$, and $S_{d\pi}$ of appropriate size satisfying the following LMIs simultaneously:

$$\begin{bmatrix} S(\theta^{[l]}) & B_{xw}^\top\left(\theta^{[l]}\right) \\ B_{xw}\left(\theta^{[l]}\right) & P_d(\theta^{[l]}) \end{bmatrix} \succ 0$$
$$\text{diag}\left\{P_d(\theta_+^{[l]}), \quad 0, \quad -P_d(\theta^{[l]}), \quad -I\right\}$$
$$\prec \left\{\begin{bmatrix} I \\ 0 \end{bmatrix} N_{dgx}\left(\theta^{[l]}\right)\right\}^s + \left\{S_{d\pi}N_{dg\pi}\left(\theta^{[l]}\right)\right\}^s \tag{38}$$

then the gain is $K(\theta) = \sum_{l=(1,\ldots,1)}^{N_\theta=(v_1,\ldots,v_N)} \xi_{1,l_1}\cdots\xi_{N,l_N}K(\theta^{[l]})$, with $K(\theta^{[l]}) = S_{dy}^\top(\theta^{[l]})(S_{dx}^\top(\theta^{[l]}))^{-1}$, which satisfies the H_2 performance for the closed-loop system whatever $\theta \in \Theta$.

Since the proof of Theorem 5 can be proved similarly as Theorem 4, the details are omitted here for brevity.

Remark 7: It follows from (22), (30), and (31) that the sizes of the matrices $P_d(\theta^{[l]})$, $P_d(\theta_+^{[l]})$, $S(\theta^{[l]})$, S_{dx}, S_{dy}, and $S_{d\pi}$ to be solved by the LMIs of (31) are determined by the dimension of the DMAR model. Thus, the complexity of the LMI problems solved by Theorem 4 is directly related to the dimension of the DMAR model. To highlight this point, the number of decision variables N_v and the total number of rows in all LMI restrictions N_r for the LMIs of Theorem 4 are computed. For a DMAR model with dimension r, we have

$$N_v = \left[0.5 \times (m_x^2 + m_w^2 + m_x + m_w)\right] \times N_\theta + 1$$
$$+ (2m_x + m_z + r) \times r + m_x^2 + m_x m_u \tag{39a}$$
$$N_r = (3m_x + m_z + m_w + r) \times N_\theta^2 + m_x. \tag{39b}$$

Similarly, based on (22), (37), and (38), we have

$$N_v^{SG} = \left(1.5m_x^2 + 0.5m_w^2 + m_x m_u + 0.5m_x + 0.5m_w\right)$$
$$\times N_\theta + (2m_x + m_z + r) \times r + 1 \tag{40a}$$
$$N_r^{SG} = N_r^S = (3m_x + m_z + m_w + r) \times N_\theta^2 + m_x \tag{40b}$$

where N_v^{SG} is the number of decision variables and N_r^{SG} is the total number of rows in all LMI restrictions for the LMIs of Theorem 5, respectively. It can be seen from (39) and (40) that the number of decision variables and the total rows of all LMI restrictions are all directly related the dimension of DMARs. Thus, in order to reduce the complexity of the H_2 control design, it is necessary to reduce the dimension of DMARs for discrete-time LPV systems.

B. Comparisons and Numerical Examples

It should be noted that the LPV/LFR is another power description for plants subject to rational uncertainties and time-varying parameters [18], [22]. However, most existing contributions to H_2 controller design focuses primarily on continuous-time LPV/LFR systems and utilize single Lyapunov functions, which can be restrictive for obtaining robust controllers [18]. As a result, there is a lack of research addressing controller design conditions for discrete-time LPV/LFR systems, which are often more complex than their continuous-time counterparts [18]. A significant contribution to this area is presented by Apkarian et al. [23], where they provide conditions for H_2 controllers, but these are limited to quadratic conditions based on parameter-independent functions, which are known to be more conservative than parameter-dependent Lyapunov (PDL) functions. A more recent and noteworthy work is the work of Pereira and de Oliveira [18], which proposes less conservative conditions for H_2 robust controller designs for discrete-time LPV/LFR systems. This work utilizes PDL in combination with slack variables and general full-block multipliers, but it makes conservative assumptions regarding the coefficient matrices in the exogenous input vector and controlled output equations.

Specifically, the LPV/LFR system for the H_2 state-feedback control studied in [18] is given by

$$x_{k+1} = A_{xx}x_k + B_{xq}q_k + B_{xu}u_k + B_{xw}w_k$$
$$p_k = C_{pp}x_k + D_{pq}q_k + D_{pu}u_k$$
$$z_k = C_{zx}x_k + D_{zu}u_k, \; q_k = \Delta(\theta)p_k \tag{41}$$

with $\Delta(\theta) = \text{diag}\{\theta_1 I_{r_1}, \ldots, \theta_n I_{r_n}\}$. The system described by (41) can be rewritten as

$$x_{k+1} = \left(A_{xx} + B_{xq}\Delta(\theta)(I - D_{pq}\Delta(\theta))^{-1}C_{pp}\right)x_k$$
$$+ \left(B_{xu} + B_{xq}\Delta(\theta)(I - D_{pq}\Delta(\theta))^{-1}D_{pu}\right)u_k + B_{xw}w_k$$
$$z_k = C_{zx}x_k + D_{zu}u_k. \tag{42a}$$

Compared with (22) in the current study, it can be observed that the coefficient matrices C_{zx} and D_{zu} in the controlled output equation, as well as the matrix B_{xw} in the exogenous input vector, are restricted to constant matrices in (42). In contrast, the system in (22) allows for a wider range of configurations, where the matrix B_{xw} is multiaffine and the matrices C_{zx} and D_{zu} in the controlled output equation are rational functions of the parameters. The similar advantage can be derived for the gain-scheduling controllers.

Example 3: In order to show the effectiveness of the proposed methods in time-varying parameter systems, consider the discrete-time

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TABLE I
NUMERICAL RESULTS FOR H_2 STATE-FEEDBACK CONTROL OF DIFFERENT METHODS

Methods	H_2 performance
Apkarian et al. [23]	3.7600
Lee and Park [24]	3.7600
Pereira and de Oliveira [18]	2.8540
Pereira and de Oliveira (gain-scheduled) [18]	1.7270
Theorem 4	2.8300
Theorem 5	1.7270

TABLE II
COMPARISON OF THE MAXIMUM UNCERTAINTY RANGE OF θ FOR DIFFERENT METHODS

Methods	δ
Pereira and de Oliveira [18]	0.3139
Pereira and de Oliveira (gain-scheduled) [18]	0.5303
Theorem 4	0.3150
Theorem 5	0.9919

uncertain LPV system borrowed from Apkarian et al. [23]

$$\left[A_{xx}(\theta)\mid B_{xu}(\theta)\mid B_{xw}(\theta)\right]=\left[\begin{array}{ccc|c|cc}\theta_2+2 & 0 & 1 & \theta_1+1 & 0 & 0\\ 1 & 0.5 & 0 & 0 & 1 & 0\\ 0 & 2 & -1 & 0 & 0 & 0\end{array}\right]$$
$$\left[C_{zx}(\theta)\mid D_{zu}(\theta)\right]=\left[\begin{array}{ccc|c}\boldsymbol{e}_1 & \boldsymbol{e}_2 & \boldsymbol{e}_3 & \boldsymbol{e}_4\end{array}\right] \quad (43)$$

with uncertain parameters $\theta_1\in[-\delta_1,\delta_1]$ and $\theta_2\in[-\delta_2,\delta_2]$, where $\boldsymbol{e}_k\in\boldsymbol{R}^4$ denotes a column vector with entries being zeros except the kth entry being 1, $k\in\{1,\ldots,4\}$.

The 4-D DMAR is given by

$$M_1(\theta)^\top=\left[(\theta_2+2)\boldsymbol{e}_1+2\boldsymbol{e}_3+(\theta_1+1)\boldsymbol{e}_4\quad \boldsymbol{e}_1+\boldsymbol{e}_2\quad 2\boldsymbol{e}_2-\boldsymbol{e}_3\quad \boldsymbol{e}_1\quad 2\boldsymbol{e}_2\quad 2\boldsymbol{e}_3\quad \boldsymbol{e}_4\right]$$
$$M_2(\theta)=\left[\boldsymbol{e}_1\quad 2\boldsymbol{e}_2\quad 2\boldsymbol{e}_3\quad \boldsymbol{e}_4\right],\; M_3(\theta)=I_4. \quad (44)$$

For state-feedback controller, according to Theorem 4, taking $\delta_1=\delta_2=0.3$, a feasible solution with $\mu_d=2.8300$ has been achieved, and the state-feedback gain is $K=[-2.1016\quad -0.3316\quad -0.8594]$.

The comparison with the representative methods, including Pereira and de Oliveira [18], Apkarian et al. [23], and Lee and Park [24], are given in Table I . It can be seen that the state-feedback controller proposed in Theorem 4 is less conservative than the representative methods, and the H_2 gain-scheduled controllers proposed in Theorem 5 and [18] are less conservative than the other methods. Moreover, it can be seen from comparison of the maximum uncertainty ranges in Table II that Theorem 5 is less conservative than the other methods.

C. H_2 Control Designs With CCIS

As explained in Remark 1, the existing LFR realization methods [15], [16] often result in high-dimensional DMARs, which increase the cost of subsequent H_2 feedback controller synthesis, or may even make the feedback controller synthesis impossible due to limited computing resources. Moreover, it will be shown that the H_2 performance results based on low-dimensional DMAR, which is obtained using the developed CCIS reduction technique, may be less conservative than those obtained directly from the original high-dimensional DMAR.

Example 4: In order to show the advantages of employing the efficient CCIS method in the design of H_2 feedback controllers, consider the following discrete-time systems:

$$\left[A_{xx}(\theta)\mid B_{xu}(\theta)\right]=\frac{1}{d}\left[\begin{array}{ccc|cc}a_{11} & a_{12} & a_{13} & b_{11} & b_{12}\\ a_{21} & a_{22} & a_{23} & b_{21} & b_{22}\\ a_{31} & a_{32} & a_{33} & b_{31} & b_{32}\end{array}\right],$$
$$B_{xw}(\theta)=\begin{bmatrix}1\\1\\0\end{bmatrix}$$
$$\left[C_{zx}(\theta)\mid D_{zu}(\theta)\right]=\left[\begin{array}{ccc|cc}1 & -1 & 0 & -3 & 0\\ \frac{c_{21}}{d} & \frac{c_{22}}{d} & \frac{c_{23}}{d} & \frac{d_{21}}{d} & \frac{d_{22}}{d}\end{array}\right],$$
$$C_y=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix} \quad (45)$$

with

$$d=\theta_1^2\theta_2^2\theta_3^2+8\theta_1^2\theta_2^2\theta_3-\theta_1,\; a_{11}=\theta_1^3\theta_2^2\theta_3^2+8\theta_1^3\theta_2^2\theta_3-\theta_1^3\theta_2\theta_3^2-8\theta_1^3\theta_2\theta_3+\theta_1^2\theta_2^2\theta_3^2+8\theta_1^2\theta_2^2\theta_3-\theta_1^2-\theta_1\theta_2^2\theta_3-8\theta_1\theta_2^2+8\theta_3$$
$$a_{12}=\theta_1^3\theta_2^2\theta_3^2+8\theta_1^3\theta_2^2\theta_3-\theta_1^3\theta_2\theta_3^2-8\theta_1^3\theta_2\theta_3-\theta_1^2\theta_2^2\theta_3^2-8\theta_1^2\theta_2^2\theta_3-\theta_1^2+\theta_1\theta_2^2\theta_3+8\theta_1\theta_2^2-8\theta_3,\; a_{13}=8\theta_1^2\theta_2+\theta_1^2\theta_2\theta_3-8\theta_1$$
$$a_{21}=-\theta_1^3\theta_2\theta_3^2+2\theta_1^2\theta_2^2\theta_3^2+8\theta_1^2\theta_2^2\theta_3-\theta_1\theta_2^2\theta_3^4+2\theta_1\theta_2^2\theta_3^2+15\theta_1\theta_2^2\theta_3-\theta_1+\theta_3^2-2,\; a_{23}=\theta_1^2\theta_2^2\theta_3^3+\theta_1^2\theta_2\theta_3-\theta_1\theta_3$$
$$a_{22}=-\theta_1^3\theta_2\theta_3^2+8\theta_1^2\theta_2^2\theta_3+\theta_1\theta_2^2\theta_3^4-2\theta_1\theta_2^2\theta_3^2-15\theta_1\theta_2^2\theta_3-\theta_1-\theta_3+2$$
$$a_{33}=8\theta_1^2\theta_2\theta_3^2-\theta_1^2\theta_3,\; b_{32}=9\theta_1^2\theta_2\theta_3^2$$
$$a_{31}=\theta_1^3\theta_3^2-\theta_1^2\theta_2\theta_3^2+\theta_1\theta_2^2\theta_3^2+8\theta_1\theta_2^2\theta_3-8\theta_1\theta_2\theta_3^3+\theta_1\theta_2\theta_3-1$$
$$a_{32}=\theta_1^3\theta_3^2+\theta_1^2\theta_2\theta_3^2-\theta_1\theta_2^2\theta_3^2-8\theta_1\theta_2^2\theta_3+8\theta_1\theta_2\theta_3^3-\theta_1\theta_2\theta_3+1$$
$$b_{11}=\theta_1^3\theta_2^2\theta_3^2+16\theta_1^3\theta_2^2\theta_3-2\theta_1^3\theta_2\theta_3^2-16\theta_1^3\theta_2\theta_3-3\theta_1^2\theta_2^2\theta_3^2-24\theta_1^2\theta_2^2\theta_3-2\theta_1^2+3\theta_1\theta_2^2\theta_3+24\theta_1\theta_2^2+3\theta_1-24\theta_3,$$
$$b_{12}=-9\theta_1$$
$$b_{21}=-2\theta_1^3\theta_2\theta_3^2+2\theta_1^2\theta_2^2\theta_3^2+16\theta_1^2\theta_2^2\theta_3+3\theta_1\theta_2^2\theta_3^4-6\theta_1\theta_2^2\theta_3^2-45\theta_1\theta_2^2\theta_3-2\theta_1-3\theta_3^2+6,\; b_{22}=\theta_1^2\theta_2^2\theta_3^3-\theta_1^2\theta_2^2\theta_3^2-\theta_1-\theta_3$$
$$b_{31}=2\theta_1^3\theta_3^2-3\theta_1\theta_2^2\theta_3^2-24\theta_1\theta_2^2\theta_3+24\theta_1\theta_2\theta_3^3-3\theta_1\theta_2\theta_3+3$$
$$c_{21}=-\theta_1^3\theta_2^2\theta_3^2-\theta_1^3\theta_2\theta_3^2-8\theta_1^3\theta_2\theta_3+\theta_1^2\theta_2^3\theta_3^2+\theta_1^2\theta_2^2\theta_3^3+8\theta_1^2\theta_2^2\theta_3^2-\theta_1\theta_2^3\theta_3^4-\theta_1\theta_2^3\theta_3-\theta_1\theta_2^2\theta_3-8\theta_1\theta_2^2-\theta_1\theta_3+\theta_1+\theta_2\theta_3^2+8\theta_3$$
$$c_{22}=-\theta_1^3\theta_2^2\theta_3^2-\theta_1^3\theta_2\theta_3^2-8\theta_1^3\theta_2\theta_3-\theta_1^2\theta_2^3\theta_3^2+\theta_1^2\theta_2^2\theta_3^3+8\theta_1^2\theta_2^2\theta_3^2+\theta_1\theta_2^3\theta_3^4+\theta_1\theta_2^3\theta_3+\theta_1\theta_2^2\theta_3+8\theta_1\theta_2^2-\theta_1\theta_3-\theta_1-\theta_2\theta_3^2-8\theta_3$$
$$c_{23}=\theta_1^2\theta_2^3\theta_3^3+\theta_1^2\theta_2^2\theta_3+\theta_1^2\theta_2\theta_3+8\theta_1^2\theta_2-\theta_1\theta_2\theta_3-8\theta_1$$
$$d_{22}=\theta_1^2\theta_2^3\theta_3^3-\theta_1^2\theta_2^3\theta_3^2-\theta_1\theta_2\theta_3-9\theta_1$$
$$d_{21}=-2\theta_1^3\theta_2^2\theta_3^2-2\theta_1^3\theta_2\theta_3^2-16\theta_1^3\theta_2\theta_3+2\theta_1^2\theta_2^2\theta_3^3+16\theta_1^2\theta_2^2\theta_3^2+3\theta_1\theta_2^3\theta_3^4+3\theta_1\theta_2^3\theta_3+3\theta_1\theta_2^2\theta_3+24\theta_1\theta_2^2-2\theta_1\theta_3-3\theta_2\theta_3^2-24\theta_3$$

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TABLE III
COMPARISON OF COMPUTATIONAL COMPLEXITY OF DIFFERENT METHODS

Methods	dimensions	N_r	N_v	H_2 performance	Computer time(s)
Theorem 4	12	1539	312	-	1.72
	5	1091	137	1.1065	1.03
Theorem 5	12	1539	417	1.8020	2.06
	5	1091	242	0.5167	0.98

with uncertain parameters $\theta_1 \in [-\delta_1, \delta_1]$ and $\theta_2 \in [-\delta_2, \delta_2]$. It should be noted that the coefficient matrices C_{zx} and D_{zu} in the control output equation of system (45) contain the time-varying parameter θ, so the theorems in [18] cannot deal with the H_2 control design issue for such a case.

For the DTRPS in (45), we have a 12-D DMAR in (20). To reduce the complexity of the H_2 control design, applying the proposed CCIS to DMAR in (20) yields the DMAR model in (21) with dimension 5.Table III details the total number of rows N_r, the number of scalar decision variables N_v, the performance H_2, and the computer time in all LMI constraints. As can be seen in Table III, the decision variables required after model reduction are extremely decreased to only 43.9% and 58.0% of the decision variables before model reduction in Theorems 4 and 5, respectively. The total number of rows required after model reduction is diminished to only 70.9% and 70.9% of the total number of rows before model reduction in Theorems 4 and 5, respectively. The computer time for the LMI problem of the 5-D DMAR model is almost half of that of the 12-D DMAR model.

The H_2 performance results in Table III demonstrate that for this example, the low-dimensional DMAR obtained by the proposed CCIS method leads to LMI results that are less conservative compared to those based on the original high-dimensional DMAR. This result is counterintuitive since the LMIs for the reduced-size DMAR contain fewer degrees of freedom (fewer decision variables). The reasons for this conservatism reduction deserve further investigation. Moreover, as showed in the lines of Theorem 4 that an H_2 feedback controller can still be successfully designed using the lower dimensional DMAR, which is an exact reduction of the original model by the proposed CCIS method, even when a feasible solution cannot be derived from the high-dimensional DMAR.

V. CONCLUSION

A new necessary and sufficient condition for the existence of CCIS has been established, which requires significantly less memory storage. Based on the efficient computation of CCIS, the reducibility conditions have been developed for DMARs of rational parameter systems. In addition, an H_2 feedback controller synthesis has been presented for rational parameter systems. Experiments have been given to show that the low-dimensional DMAR model obtained by CCIS reduction method may be less conservative, which further demonstrate the superiority of the established efficient calculation of CCIS.

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